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Constructing Phantom With a Nonminimally Coupled Complex Scalar Field

You-Gen Shen[1,2,3] and Xian-Hui Ge[1,2,4,5]

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In view of the recent observation data indicating that the equation of state of the dark energy might be smaller than −1, this leads to introduction of phantom models featured by their negative kinetic energy to account for the regime of equation of state $w < -1$. In this paper, we discuss the possibility of using a nonminimally coupled complex scalar field as phantom to realize the equation-of-state parameter $w < -1$. The main equations which govern the evolution of the universe are obtained. The relations between the potential of the field and red-shift, namely, the reconstruction equations are derived.

KEY WORDS: phantom; nonminimally coupled; equation-of-state parameter.

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1. INTRODUCTION

Recent astrophysical data indicate to the acceleration of the scale factor of the observable universe resulting from dark energy that has negative pressure. Many candidates for dark energy have been proposed so far to fit the current observations. Among these models, the most important ones are cosmological constant and a time varying scalar field evolving in a specific potential, referred to as "quintessence" (Caldwell *et al.*, 1998; Coble *et al.*, 1997; Gao and Shen, 2002; Gu and Hwang, 2001; Ratra and Peebles, 1988; Steinhardt *et al.*, 1999; Zlatev *et al.*, 1999), which confines the range of the equations of state parameter within $-1 < w < -1/3$. However, another simple approach to model such an accelerating scale is to consider a phantom field with negative kinetic energy that

[1] Shanghai Astronomical Observatory, Chinese Academy of Sciences, Shanghai, People's Republic of China.
[2] National Astronomical Observatories, Chinese Academy of Sciences, Beijing, People's Republic of China.
[3] Institute of Theoretical physics, Chinese Academy of Sciences, Beijing, People's Republic of China.
[4] Graduate School of Chinese Academy of Sciences, Beijing, People's Republic of China.
[5] To whom correspondence should be addressed at Shanghai Astronomical Observatory, Chinese Academy of Sciences, Shanghai 200030, People's Republic of China; e-mail: gexh@shao.ac.cn.

can realize the $w < -1$ in its evolution (Arbey *et al.*, 2001; Boiseau *et al.*, 2000; Boyle *et al.*, 2002; Caldwell, 2002; Caldwell *et al.*, 2003; Carroll *et al.*, 2003; Chiba *et al.*, 2000; Faraoni, 2002; Frampton, 2003; Maor *et al.*, 2002; Onemli and Woodard, 2002; Parker and Raval, 1999; Sahni and Starobinsky, 2002; Schulz and White, 2001; Torres, 2002), because current analysis to the observation data indicates that the range of the equation of state may not always be greater than -1, in fact, they can lie in the range $-1.38 < w < -0.82$ (Melchiorri *et al.*, 2003). Matter with $w < -1$, has received increasing attention among theorists recently. It has some strange properties, for example, the energy density of phantom energy increases with time. It also violates the dominant-energy condition (Gibbons, 2003; Hawking and Ellis, 1973), which helps prohibit time machines and wormholes. However, phantom is an intriguing topic because it fits current observations.

Galdwell has shown that if the dark energy is phantom energy, our universe would end in a *Big Rip* (Caldwell, 2002; Caldwell *et al.*, 2003). Theorists have already proposed several scalar-field models for phantom energy (Armendariz-Picon *et al.*, 1999; Caldwell, 2002; Carroll and Guica, 2003; Parker and Raval, 1999; Sahni and Starobinsky, 2002; Schulz and White, 2001; Torres, 2002). Stringy phantom energy (Frampton, 2003) and brane-world phantom energy (Frampton and Shtanov, 2003) have also been discussed. In this paper, we wish to point out the possibility of using a nonminimally coupled complex scalar field. We also discuss the feasibility of yielding the equation of the state of phantom with the data $r(z)$, for the information on phantom may be determined with the observation data $r(z)$ from the reconstruction equations.

The paper proceeds as follows. First, we will show the condition for the nonminimally coupled complex scalar field to be phantom. Then starting from Einstein equations we obtain the main equations governing the evolution of universe and then we present numerical analysis results of state equations and energy density equation. From the main equations we obtain the reconstruction equations in which the phantom is related to the observation quantities. Finally, a brief discussion is given. Throughout the paper, the units $G = c = 1$ are used.

2. THE MODEL

The flat Robertson–Walker metric is given by

$$ds^2 = dt^2 - a^2(t)\left(dr^2 + r^2 d\varphi_1^2 + r^2 \sin^2\varphi_1\, d\varphi_2^2\right), \tag{1}$$

where $a(t)$ is the scale factor of the universe.

The Lagrangian density for the nonminimally coupled complex scalar field Φ which is nonminimally coupled to the curvature is

$$\mathcal{L} = \sqrt{-g}\left[-\frac{1}{2}\partial_\mu\Phi\partial^\mu\Phi^* - \frac{1}{2}\xi R\Phi\Phi^* - V(|\Phi|)\right], \tag{2}$$

where ξ is a numerical factor, V is the potential of the field, and $R = 6(a\ddot{a} + \dot{a}^2)/a^2$ is the Ricci scalar. Here we would like to write the field in the form

$$\Phi(t) = \phi(t)\, e^{i\theta(t)}, \tag{3}$$

where $\phi(t)$ and $\theta(t)$ are the amplitude and the phase of the field, respectively. Then the Lagrangian density becomes

$$\mathcal{L} = \sqrt{-g}\left[-\frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{2}\phi^2\partial_\mu\theta\partial^\mu\theta - \frac{1}{2}\xi R\phi^2 - V(\phi)\right]. \tag{4}$$

Now we decompose the field into homogeneous parts and fluctuations as follows

$$\Phi = [\phi(t) + \delta\phi(t, \vec{x})]e^{i[\theta(t)+\delta\theta(t,\vec{x})]}. \tag{5}$$

Using Equation (4), the Lagrangian can be written as

$$\begin{aligned}\mathcal{L} = \sqrt{-g}\Bigg\{&-\frac{1}{2}(\dot{\phi} + \delta\dot{\phi})^2 + \frac{1}{2a^2}(\nabla\delta\phi)^2 - \frac{1}{2}\xi R(\phi + \delta\phi)^2 \\ &-\frac{1}{2}(\phi + \delta\phi)^2\left[\left(\dot{\theta} + \delta\dot{\theta}\right)^2 - \frac{1}{a^2}(\nabla\delta\theta)^2\right] - V(\phi + \delta\phi)\Bigg\},\end{aligned} \tag{6}$$

where "·" denotes the derivative with respect to t, ∇ is the Laplace operator.

Assuming the gravity effects are weak, which is a good approximation here, we obtain the equation of motion of the field (Boyle *et al.*, 2002; Frampton, 2003)

$$\ddot{\phi} + 3H\dot{\phi} - \dot{\theta}^2\phi - \xi R\phi - V' = 0, \tag{7}$$

$$\phi\ddot{\theta} + 3H\phi\dot{\theta} + 2\dot{\phi}\dot{\theta} = 0, \tag{8}$$

for the homogeneous parts and

$$\delta\ddot{\phi} + 3H\delta\dot{\phi} - 2\dot{\theta}\phi\delta\dot{\theta} - \dot{\theta}^2\delta\phi - \frac{\nabla^2}{a^2}\delta\phi - \xi R\delta\phi - V''(\phi)\,\delta\phi = 0, \tag{9}$$

$$\phi\delta\ddot{\theta} + 3H\phi\delta\dot{\theta} + 2\dot{\phi}\delta\dot{\theta} + 2\dot{\theta}\delta\dot{\phi} - 2\frac{\dot{\phi}}{\phi}\dot{\theta}\delta\phi - \phi\frac{\nabla^2}{a^2}\delta\theta = 0, \tag{10}$$

for fluctuations.

The solution can be obtained in the following form

$$\delta\phi = \delta\phi_0 e^{\alpha(t)+i\vec{k}\cdot\vec{x}}, \quad \delta\theta = \delta\theta_0 e^{\alpha(t)+i\vec{k}\cdot\vec{x}}. \tag{11}$$

From the above, we can see that the fluctuations will grow exponentially and go nonlinear to form Q balls if α is real and positive. Substituting Equations (11) into

Equations (9) and (10), we obtain

$$\left(\ddot{\alpha} + \dot{\alpha}^2 + 3H\dot{\alpha} + \frac{k^2}{a^2} - V'' - \xi R - \dot{\theta}^2\right)$$
$$\times \left(\ddot{\alpha} + \dot{\alpha}^2 + 3H\dot{\alpha} + \frac{k^2}{a^2} + \frac{2\dot{\phi}\dot{\alpha}}{\phi}\right) + 4\dot{\theta}^2\dot{\alpha}^2 = 0. \quad (12)$$

It can be simplified to be

$$\dot{\alpha}^4 + \left(2\frac{k^2}{a^2} + V'' + \xi R + 3\dot{\theta}^2\right)\dot{\alpha}^2 + \left(\frac{k^2}{a^2} - V'' - \xi R + \dot{\theta}^2\right)\frac{k^2}{a^2} = 0, \quad (13)$$

where the cosmological expansion is neglected. So the orbit of the field in the potential is circular. We also assume that $\ddot{\alpha} \ll \dot{\alpha}^2$. In order for α to be real and positive, we should have the last term of Equation (13) to be negative. Therefore, we obtain the instability band

$$0 < \frac{k^2}{a^2} < -\dot{\theta}^2 + V'' + \xi R. \quad (14)$$

Since the curvature of the potential is negative for $w < -1$, the stability band always doesn't exist for the so-called minimally coupled case, $\xi = 0$. In the coupled case, ξR is positive, the stability condition can be written as follows

$$\dot{\theta}^2 - V'' < \xi R. \quad (15)$$

Therefore, the nonminimally coupled parameter ensures the stability of the field.

The Einstein equations can be written as

$$3\frac{\dot{a}^2}{a^2} = 8\pi\left[\rho_m - \frac{1}{2}\dot{\phi}^2 + V - \frac{1}{2}\phi^2\dot{\theta}^2 + \frac{3}{2}\xi^2 R\phi^2\right.$$
$$\left. + \xi\left(-\frac{3}{2}\phi\ddot{\phi} + \frac{3}{2}H\phi\dot{\phi} + 3H^2\phi^2 + 3V\right)\right], \quad (16)$$

$$\frac{2a\ddot{a} + \dot{a}^2}{a^2} = 8\pi\left[\frac{1}{2}\dot{\phi}^2 + V + \frac{1}{2}\phi^2\dot{\theta}^2 + \frac{3}{2}\xi^2 R\phi^2\right.$$
$$\left. + \xi\left(2\dot{\phi}^2 + \frac{1}{2}\phi\ddot{\phi} + \frac{3}{2}H\phi\dot{\phi} - \phi^2 G_1^1 + 3V\right)\right]. \quad (17)$$

Where G_1^1 is Einstein tensor. The coupled complex scalar field contributes the energy density ρ_Φ and pressure p_Φ as follows

$$\rho_\Phi = -\frac{1}{2}\dot{\phi}^2 + V - \frac{1}{2}\phi^2\dot{\theta}^2 + \frac{3}{2}\xi^2 R\phi^2$$

$$+\xi\left(-\frac{3}{2}\phi\ddot{\phi}+\frac{3}{2}H\phi\dot{\phi}+3H^2\phi^2+3V\right), \tag{18}$$

$$\begin{aligned}p_\Phi &= -\frac{1}{2}\dot{\phi}^2 - V - \frac{1}{2}\phi^2\dot{\theta}^2 - \frac{3}{2}\xi^2 R\phi^2 \\ &\quad -\xi\left(2\dot{\phi}^2-\frac{1}{2}\phi\ddot{\phi}+\frac{3}{2}H\phi\dot{\phi}-\phi^2G_1^1+3V\right).\end{aligned} \tag{19}$$

Equations (7), (8), (16), and (17) are the main equations governing the evolution of the universe. If we focus on the strongly coupled case, the equation-of-state for phantom fields is

$$w=\frac{p_\Phi}{\rho_\Phi}=\frac{\dot{\phi}^2+2V+\phi^2\dot{\theta}^2+3\xi^2R\phi^2}{\dot{\phi}^2-2V+\phi^2\dot{\theta}^2-3\xi^2R\phi^2}.$$

It is clear that the strongly coupled phantom field could realize the equation-of-state parameter $w<-1$, which is equivalent to

$$0<\dot{\phi}^2+\phi^2\dot{\theta}^2<2V(\phi)+3\xi^2R\phi^2, \tag{20}$$

where the term $\phi^2\dot{\theta}^2$ comes from the "total angular motion." The most prominent feature of the coupled phantom fields is that it will not reduce to a cosmological constant even when the ϕ is spatially uniform and time-independent. The solution of Equation (8) can be written as

$$\dot{\theta}=\frac{c}{a^3\phi^2}, \tag{21}$$

where c is an integration constant. Inserting Equation (22) into Equations (7), (16), and (17) we get

$$\ddot{\phi}+3H\dot{\phi}-\frac{d}{d\phi}\left(\frac{c^2}{2a^6\phi^2}+\frac{1}{2}\xi R\phi^2+V\right)=0, \tag{22}$$

$$\begin{aligned}H^2=\frac{\dot{a}^2}{a^2}&=\frac{8\pi}{3}\left[\rho_m-\frac{1}{2}\dot{\phi}^2+V-\frac{c^2}{2a^6\phi^2}+\frac{3}{2}\xi^2R\phi^2\right.\\ &\quad\left.+\xi\left(-\frac{3}{2}\phi\ddot{\phi}+\frac{3}{2}H\phi\dot{\phi}+3H^2\phi^2+3V\right)\right],\end{aligned} \tag{23}$$

$$\begin{aligned}\frac{\ddot{a}}{a}&=-\frac{4\pi}{3}\left[\rho_m-2\dot{\phi}^2-2V-\frac{2c^2}{a^6\phi^2}-3\xi^2R\phi^2\right.\\ &\quad\left.+\xi\left(-6\dot{\phi}^2-3\ddot{\phi}\phi-3H\phi\dot{\phi}+6\frac{\ddot{a}}{a}\phi^2-6V\right)\right],\end{aligned} \tag{24}$$

where H is the Hubble parameter. The term $c^2/\left(2a^6\phi^2\right)$ and $\frac{1}{2}\xi R\phi^2$ in Equation (23) which are coming from the "angular motion" and the "coupled effect" of the field can be teated as an effective potential. It produces a "centrifugal force" and tends to drive ϕ away from zero. Here we focus on the strongly coupled case in Equations (23) and (24). They can be simplified to be

$$H^2 = \frac{\dot{a}^2}{a^2} = \frac{8\pi}{3}\left[\rho_m - \frac{1}{2}\dot{\phi}^2 + V - \frac{c^2}{2a^6\phi^2} + \frac{3}{2}\xi^2 R\phi^2\right], \tag{25}$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi}{3}\left[\rho_m - 2\dot{\phi}^2 - 2V - \frac{2c^2}{a^6\phi^2} - 3\xi^2 R\phi^2\right]. \tag{26}$$

In Equations (25) and (26), the contributions from the angular motion and coupled effect to the energy density and pressure are proportional to $a^{-6}\phi^{-2}$ and $R\phi^2 \sim a^{-3}\phi^2$ (R is like the matter density $\rho_m \propto a^{-3}$), respectively. Provided that ϕ decreases faster than $a^{-3/2}$, then the contribution of angular motion becomes dominant, while that of the coupled effect can be neglected. On the other hand, the factor a^{-6} in the angular motion may make these contributions decrease very fast, even faster than the matter density $\rho_{\rm m}$ (i.e., a^{-3}), provided that ϕ does not decrease as fast as $a^{-3/2}$. And contributions from the coupled effect are always less than that of the matter density $\rho_{\rm m}$, as long as ϕ decreases.

3. DYNAMICAL EVOLUTION OF PHANTOM FIELDS

In the following, we would like to investigate the global structure of the dynamical system and compute the cosmological evolution by numerical analysis. Thus, we rewrite the equations of motion as

$$\begin{aligned}\dot{H} = &-\frac{\lambda^2}{6(1-\lambda^2\xi\phi^2)}\left[\rho_\gamma - 2\dot{\phi}^2 - 2V - \frac{2c^2}{a^6\phi^2} - 3\xi^2 R\phi^2\right.\\ &\left.+\xi\left(-6\dot{\phi}^2 - 3\ddot{\phi}\phi - 3H\phi\dot{\phi} + 6\frac{\ddot{a}}{a}\phi^2 - 6V\right)\right]\\ &-\frac{\lambda^2}{3}\left[\rho_\gamma - \frac{1}{2}\dot{\phi}^2 + V - \frac{c^2}{2a^6\phi^2} + \frac{3}{2}\xi^2 R\phi^2\right.\\ &\left.+\xi\left(-\frac{3}{2}\phi\ddot{\phi} + \frac{3}{2}H\phi\dot{\phi} + 3H^2\phi^2 + 3V\right)\right],\end{aligned} \tag{27}$$

$$\dot{\rho_\gamma} = -3H(\rho_\gamma + p_\gamma), \tag{28}$$

$$\ddot{\phi} + 3H\dot{\phi} - \frac{d}{d\phi}\left(\frac{c^2}{2a^6\phi^2} + \frac{1}{2}\xi R\phi^2 + V(\phi)\right) = 0, \tag{29}$$

$$H^2 = \frac{\lambda^2}{3}\left[\rho_\gamma - \frac{1}{2}\dot{\phi}^2 + V - \frac{c^2}{2a^6\phi^2} + \frac{3}{2}\xi^2 R\phi^2 \right.$$
$$\left. + \xi\left(-\frac{3}{2}\phi\ddot{\phi} + \frac{3}{2}H\phi\dot{\phi} + 3H^2\phi^2 + 3V\right)\right], \tag{30}$$

where the potential $V(\phi)$ is exponentially dependent on ϕ as $V(|\Phi|) = V_0 \exp(-\eta\lambda|\Phi|)$, $\lambda^2 = 8\pi$, and η is a constant. Now we can introduce the following variables to obtain the autonomous system for the above dynamical system. The variables are defined as

$$x = \frac{\lambda\dot{\phi}}{\sqrt{6}H},\ y = \frac{\lambda\sqrt{V(\phi)}}{\sqrt{3}H},\ z = \frac{\lambda c}{\sqrt{6}Ha^3\phi},\ \chi = \frac{1}{\lambda\phi},\ u = \frac{\lambda\phi^3}{H},\ N = \ln a(t)$$

then Equations (27)–(30) can be written as

$$\frac{dx}{dN} = \frac{\lambda\ddot{\phi}}{\sqrt{6}H} - \frac{\lambda\dot{\phi}\dot{H}}{\sqrt{6}H^3}, \tag{31}$$

$$\frac{dy}{dN} = \frac{\lambda V'(\phi)\dot{\phi}}{2H^2\sqrt{3V(\phi)}} - \frac{\lambda\sqrt{V(\phi)}\dot{H}}{\sqrt{3}H^3}, \tag{32}$$

$$\frac{dz}{dN} = -\frac{\lambda c\dot{\phi}}{\sqrt{6}Ha^3\phi^2} - \frac{3\lambda c}{\sqrt{6}a^3\phi} - \frac{\lambda c\dot{H}}{\sqrt{6}H^3a^3\phi}, \tag{33}$$

$$\frac{du}{dN} = \frac{3\phi^2\dot{\phi}}{H} - \frac{\phi^3\dot{H}}{H^2}, \tag{34}$$

$$\frac{d\chi}{dN} = -\frac{\dot{\phi}}{\lambda\phi^2 H}, \tag{35}$$

where $\ddot{\phi}$ and $\dot{H}$ are

$$\ddot{\phi} = \frac{6z^2H^2\chi}{\lambda} - \frac{3\eta y^2H^2}{\lambda} - \frac{3\sqrt{6}xH^2}{\lambda} - \frac{\sqrt{6}\xi uzH^2}{2\lambda},$$

$$\dot{H} = -\frac{\lambda^2}{6(1-\lambda^2\xi\phi^2)}(\rho_\gamma - \frac{12x^2H^2 - 6y^2H^2 - 12H^2z^2}{\lambda^2} - 3\sqrt{6}H^2uz)$$
$$+\xi\left(-\frac{36x^2H^2}{\lambda^2} - 3\left(\frac{6H^2z^2\chi - 3\eta H^2y^2 - 3\sqrt{6}xH^2}{\lambda} - \frac{\sqrt{6}\xi uzH^2}{2\lambda}\right)\frac{1}{\lambda\phi}\right.$$
$$\left. - \frac{3\sqrt{6}xH^2}{\lambda^2\chi} - \frac{18y^2H^2}{\lambda^2}\right)$$
$$-\frac{\lambda^2}{3}\left(\rho_\gamma - \frac{3x^2H^2 - 3y^2H^2 + 6z^2H^2}{\lambda^2} + \frac{3\sqrt{6}H^2uz}{2}\right)$$

$$-\frac{\lambda^2\xi}{3}\left(-\frac{3}{2}\left(\frac{6H^2z^2\chi-3\eta H^2y^2-3\sqrt{6}xH^2}{\lambda}-\frac{\sqrt{6}\xi uzH^2}{2\lambda}\right)\frac{1}{\lambda\phi}\right.$$
$$\left.+\frac{3\sqrt{6}xH^2}{2\lambda^2\chi}+\frac{3H^2}{\lambda^2\chi^2}+\frac{9y^2H^2}{\lambda^2}\right). \tag{36}$$

In the above equations, it is assumed that $R = c/a^3$. In order to make it convenient to obtain the numerical results, we only consider strong coupled scalar fields and that Equations (31)–(35) can be reduced to

$$\frac{dx}{dN}=\frac{3}{2}x\left[\gamma\left(1+x^2-y^2+z^2-\frac{3\xi^2uz}{\sqrt{6}}\right)-2(x^2+z^2)\right]$$
$$-\left(3x-\sqrt{6}z^2\xi+\sqrt{\frac{3}{2}}\eta y^2+\frac{1}{2}\xi uz\chi\right) \tag{37}$$

$$\frac{dy}{dN}=\frac{3}{2}y\left[\gamma\left(1+x^2-y^2+z^2-\frac{3\xi^2uz}{\sqrt{6}}\right)-2(x^2+z^2)\right]-\sqrt{\frac{3}{2}}\eta xy \tag{38}$$

$$\frac{dz}{dN}=-3z-\frac{3}{2}z\left[\gamma\left(1+x^2-y^2+z^2-\frac{3\xi^2uz}{\sqrt{6}}\right)-2(x^2+z^2)\right]-\sqrt{6}xz\xi \tag{39}$$

$$\frac{du}{dN}=3\sqrt{6}xu\chi-\frac{3}{2}y\left[\gamma\left(1+x^2-y^2+z^2-\frac{3\xi^2uz}{\sqrt{6}}\right)-2(x^2+z^2)\right] \tag{40}$$

$$\frac{d\chi}{dN}=-\sqrt{6}\xi^2x \tag{41}$$

And also, we have a constraint equation

$$\Omega_\phi+\frac{\kappa^2\rho_\gamma}{3H^2}=1, \tag{42}$$

where

$$\Omega_\phi=\frac{\kappa^2\rho_\phi}{3H^2}=y^2-x^2-z^2+\frac{3\xi^2uz}{\sqrt{6}} \tag{43}$$

The equation of state for the nonminimally coupled complex scalar field could be expressed in terms of the new variables as

$$w_\Phi=\frac{p_\Phi}{\rho_\Phi}=\frac{x^2+y^2+z^2+\frac{3\xi^2uz}{\sqrt{6}}}{x^2-y^2+z^2-\frac{3\xi^2uz}{\sqrt{6}}}. \tag{44}$$

The critical points of the above autonomous system are easily obtained by setting the right-hand sides of the above equations to zero. One can write the variable near the critical points $x_c, y_c, z_c, u_c, \chi_c$ in the form $x = x_c + \mu$, $y = y_c + \nu$, $z = z_c + \iota$, $u = u_c + \varsigma$, and $\chi = \chi_c + o$, where $\mu, \nu, \iota, \varsigma, o$ are perturbations of the variables near the critical points and form a column vector denoted as U. Substituting the above expression into the autonomous system (37)–(43), one can obtain the equation for the perturbation up to the first order as $U' = MU$, where the prime denotes the derivative with respect to N. The coefficients of the perturbation equations from a 5×5 matrix M whose eigenvalues determine the type and stability of the critical points (Arbey *et al.*,2002; Copeland *et al.*, 1998). The only physically meaningful critical point corresponding to the autonomous system (37)–(43) is $(x, y, z, u, \chi) = (-\lambda/\sqrt{6}, \sqrt{1+\lambda^2/6}, 0, 0, 0)$, which corresponds to the eigenvalues $(0, -3-\lambda^2/2, -3\gamma-\lambda^2, -3+\lambda^2/2, -\lambda^2/2)$. Therefore, it is a stable node of the autonomous system when $\lambda^2 < 6$. This corresponds to a late time attractor solution which is a phantom dominated epoch $\Omega_\phi = 1$ and an equation of state $w_\phi = -\lambda^2/3 - 1$. Moreover, $\lambda^2 < 6$ imposes a lower bound to the equation of state $w > -3$. The results of numerical analysis of the system are shown in Figs. 1–4, which have considered both the nonminimally coupled cases and the minimally coupled cases. From the figures, we find that the evolution of our universe in our cases seems to be influenced little by the coupled constant ξ, but is significantly affected by the parameter η, γ.

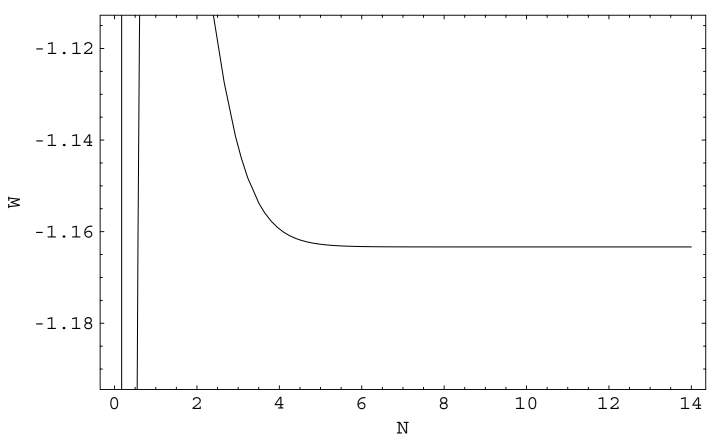

Fig. 1. The evolution of equation of state of phantom w with respect to N due to nonminimally coupled cases, for $\xi = 10$, $\eta = 0.5$,$\gamma = 0.7$.

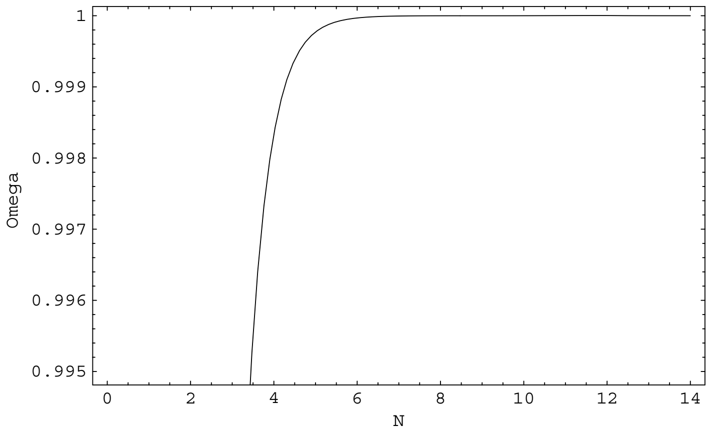

Fig. 2. The evolution of Ω_ϕ with respect to N due to nonminimally coupled cases, for $\xi = 10$, $\eta = 0.5$,$\gamma = 0.7$.

4. RECONSTRUCTION EQUATIONS

Now, we correlate the potential with the observable red-shift of SNe Ia. To do so, following the earlier study in this field, we introduce the quantity $r(z)$, H_0,

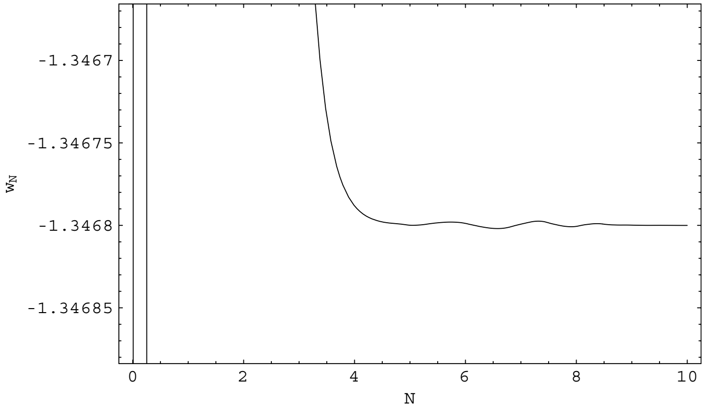

Fig. 3. The evolution of equation of state of phantom w with respect to N due to nonminimally coupled cases, for $\xi = 0$, $\eta = 1.01$,$\gamma = 1.02$.

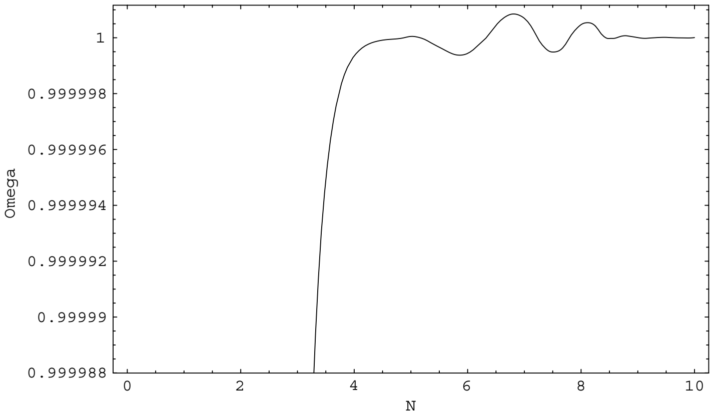

Fig. 4. The evolution of Ω_ϕ with respect to N due to nonminimally coupled cases, for $\xi = 0$, $\eta = 1.01$,$\gamma = 1.02$.

$\Omega_{\rm m}$ and $a\,(t)$, $H\,(t)$, $\rho_{\rm m}\,(t)$ as follows

$$1 + z = \frac{1}{a}, \quad r\,(z) = -\int_{t_0}^{t(z)} \frac{dt'}{a\left(t'\right)} = \int_0^z \frac{dz'}{H\left(z'\right)},$$
$$H\,(z) = \frac{\dot{a}}{a} = \frac{1}{dr/dz}, \quad \rho_m = \Omega_m \rho_c = \frac{3\Omega_m}{8\pi} H_0^2\,(1+z)^3\,, \tag{45}$$

where z, H_0, and $\Omega_{\rm m}$ are redshift, Hubble constant, and matter energy density, respectively. Using Equations (45), we get

$$\frac{\ddot{a}}{a} = \frac{1}{(dr/dz)^2} + (1+z)\,\frac{d^2r/dz^2}{(dr/dz)^3}, \tag{46}$$

together with Equations (18) and (19), we can obtain the reconstruction equations. For strongly coupled case, we have

$$V(\phi(x)) = \frac{1}{8\pi}\left[\frac{3}{(dr/dz)^2} + 2\,(1+z)\,\frac{d^2r/dz^2}{(dr/dz)^3}\right]$$
$$- \frac{3\Omega_m}{16\pi} H_0^2\,(1+z)^3 - 6\xi^2\phi^2\left[\frac{2}{(dr/dz)^2} + (1+z)\,\frac{d^2r/dz^2}{(dr/dz)^3}\right]. \tag{47}$$

$$\left(\frac{d\phi}{dz}\right)^2 + \frac{c^2}{\phi^2}\,(1+z)^4\left(\frac{dr}{dz}\right)^2 = \frac{(dr/dz)^2}{4\pi(1+z)^2}$$
$$\times\left[\frac{(1+z)\,d^2r/dz^2}{(dr/dz)^3} + \frac{3\Omega_m}{2} H_0^2\,(1+z)^3\right], \tag{48}$$

Equation (47) is the same as those ordinary quintessence while Equation (47) is different in that there is a sign difference. The right-hand side of Equation (48) is positive while it is negative in conventional nonminimally coupled complex scalar field model.

5. CONCLUSIONS

Equations (18) and (19) tell us the coupled effect makes contributions to the energy density and pressure and it is generally not negligible in the early time of the universe. However, in many cases, the conditions from the angular motion and the coupled effect of the nonminimally coupled complex scalar fields decrease very fast along with the expansion of the universe, and is negligible in the process if reconstructing the phantom potential $V(\phi)$, which is responsible for the possible accelerating expansion of the universe. Contrary to minimally coupled cases, the most important role of the nonminimally coupled parameter is to ensure the stability of the field. We also show that an attractor solution exists only if $\lambda^2 < 6$, which corresponds to a phantom energy dominant phase, which means that the universe might end in a big rip. The existence of the attractor solution $\lambda^2 < 6$ imposes a constraint on the equation of state $w > -3$.

The nature of dark energy is still a mystery now. The current data indicate that our universe is poised somewhere between quintessence, cosmological constant, and phantom energy. But the future observation, especially the longer observations by WMAP, will help in unravelling the nature of the dark energy. As soon as the equation of the state parameter $w < -1$ is confirmed by observations, the fundamental physics will be altered. And also we will face up to new cosmic fate that differs remarkably from the re-collapse or endless cooling considered before. In this paper, we have analyzed the possibility of using the nonminimally coupled complex scalar field as the phantom for accelerating the universe. We obtained the main equations which govern the evolution of the universe and rewrote them with the observable quantities. The coupled term reveals the strong action between matter and dark energy. The interaction between the two kinds of energy must be of great importance sometime in the universe evolution. In any case, the nonminimally coupled complex scalar fields as the phantom should be seriously considered since such fields have been involved in many different sectors of elementary particle physics.

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